

# Celestial Fix

## 2 LoP analytic solution

<http://sites.google.com/site/navigationalalgorithms/>

## **Index**

Variables .....	3
Equation of the Line of Position .....	3
Fix by two LoP .....	3
Running Fix .....	3
<u>Appendix</u>	
A1. Algorithm.....	5
A2. Example.....	6
A3. Software .....	6
A4. Source code .....	7
A5. References .....	7

## ***Abstract***

This paper describes the numerical solution of the problem in classical celestial navigation: calculate the position by two lines of position.

Two cases are treated: the *Fix*, where the two observations may be considered as simultaneous, and the *Running Fix*, where in general due to the time elapsed between the two measurements with the sextant, you must move the first line to the instant of the second one.

© Andrés Ruiz, 1999  
*Navigational Algorithms*  
*San Sebastián – Donostia*  
*43° 19'N 002°W*  
 Current version: 2007

## Variables

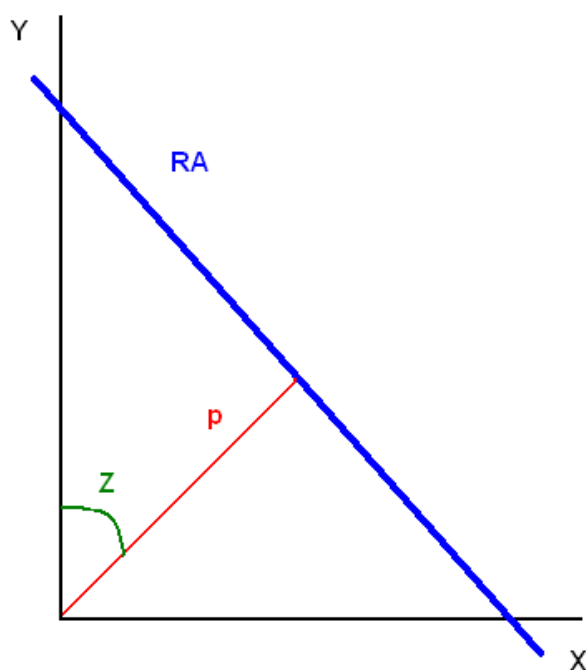
<b>P</b>	Intercept of a sight $p = H_o - H_c$ Towards = + / Away = -
<b>Z</b>	Azimuth (true) Measured clockwise around the horizon from 0° to 360°, is the arc of the horizon between the meridian of a place and the vertical circle passing through a celestial body.
<b>B</b>	Latitude N (+) / S (-)
<b>L</b>	Longitude E (+) / W (-)

## Equation of the Line of Position

The equation of a line in the Cartesian plane is given by:

$$y = f(x) = m x + b$$

Where  $m$  is the slope and  $b$  is the ordinate at the origin.



LoP in the Cartesian plane

From the figure it follows the relationship of  $m$ ,  $y$ ,  $b$ , with the parameters characterizing the line of position, LoP:  $p$  and  $Z$

$$m = \tan(180^\circ - Z) = -\tan Z$$

$$b = p / \cos Z$$

Substituting these values into the equation and rearranging terms, leads to the equation of the LoP:

$$p = x \sin Z + y \cos Z$$

## Fix by two LoP

Position ( $B$ ,  $L$ ) can be calculated analytically as the intersection of two LoP, taking the estimated position as the origin of Cartesian coordinates. The process merely mathematically formulate the graphic method used in the nautical chart.

If both are obtained simultaneously:

$$p_1 = x \sin Z_1 + y \cos Z_1$$

$$p_2 = x \sin Z_2 + y \cos Z_2$$

Solving this system of two equations the point ( $x$ ,  $y$ ) is obtained; the cut between the two LoP. The latitude and longitude will be obtained from the estimated position ( $B_e$ ,  $L_e$ ).

If  $p$  is in nautical miles:

$$B = B_e + y/60$$

$$L = L_e + x/60 / \cos(B)$$

The calculation process is detailed in the appendix. The required data is:

- Estimated Posicion:  $B_e$ ,  $L_e$
- LoP1:  $p_1$ ,  $Z_1$
- LoP2:  $p_2$ ,  $Z_2$

## Running Fix

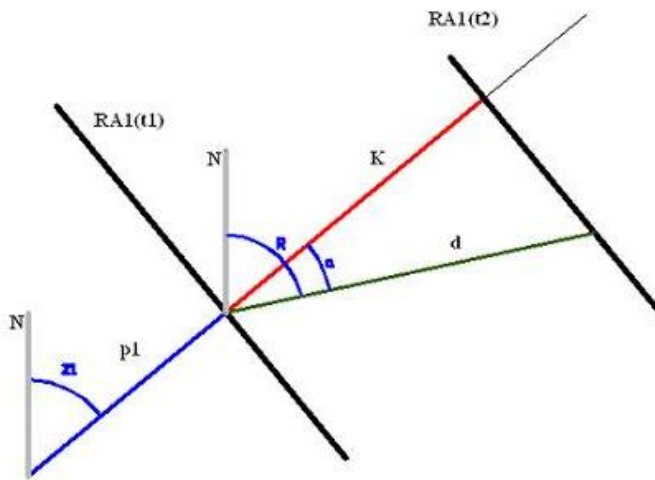
In the more general case the two LoPs will be obtained for different moments in time:  $t_1$  and  $t_2$ .

Let  $R$  and  $d$  be the course and distance sailed between  $t_1$  and  $t_2$ . The first LoP,  $RA_1(t_1)$ , must be moved to the instant of the second observation;  $RA_1(t_2)$ . In this way the intersection of  $RA_1(t_2)$  and  $RA_2(t_2)$  give us the position.

The distance sailed can be calculated as a function of time elapsed between the

observations and the speed sailed at constant course:

$$d = V (t_2 - t_1)$$



Move of a LoP.

In the figure, K is the distance to be moving the first LoP.

$$\begin{aligned}\cos \alpha &= K/d \\ Z1 + \alpha &= R \\ K &= d \cos(R - Z1)\end{aligned}$$

So the intercept was corrected by:

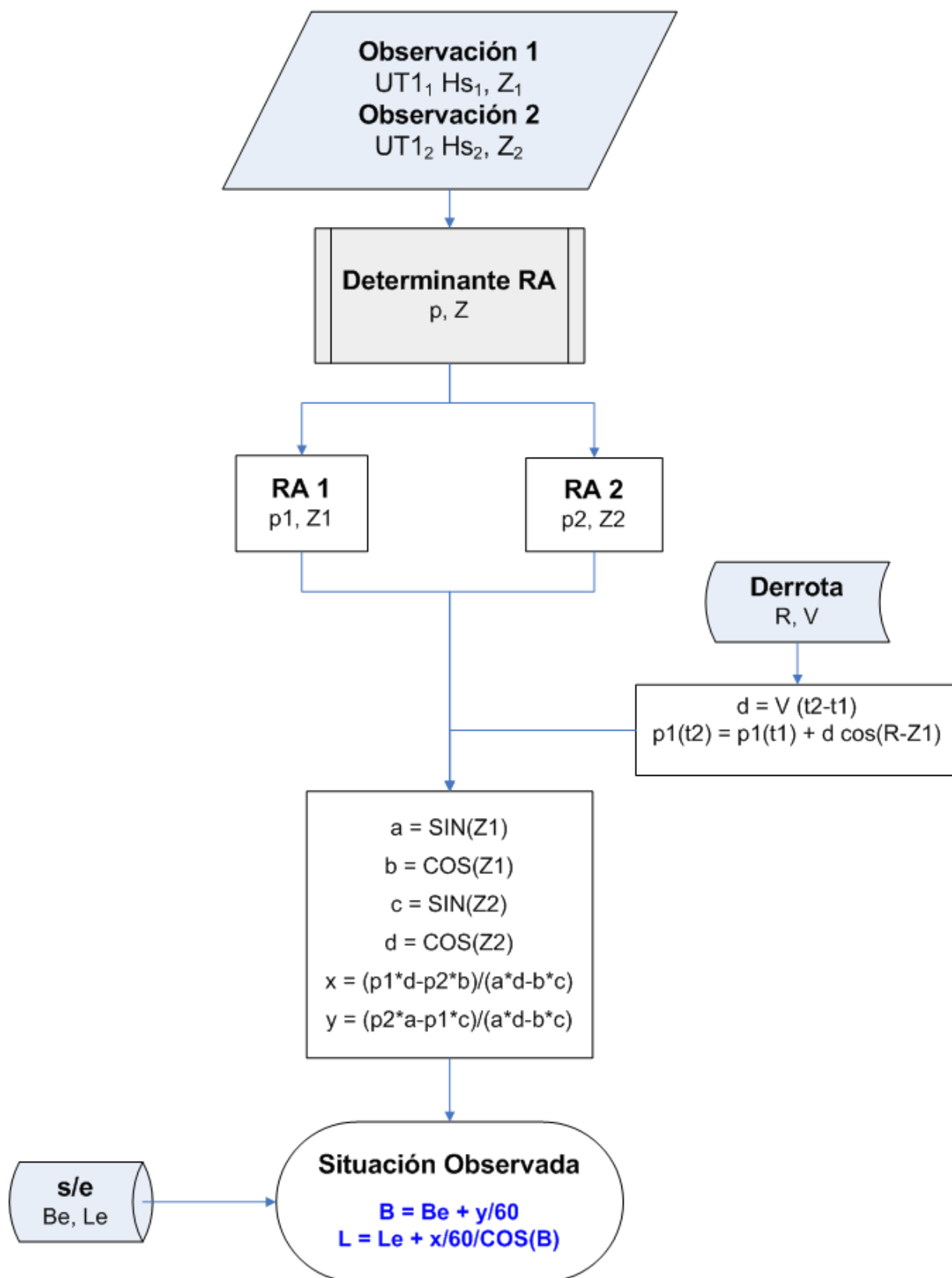
$$p1(t_2) = p1(t_1) + K$$

$$\mathbf{p1(t_2) = p1(t_1) + d \cos(R - Z1)}$$

Of course, if both observations are simultaneous,  $t_1 = t_2$ , the distance is  $d = 0$ , and the intercept does not change.

## A1. Algorithm

## Situación por dos rectas de altura



## A2. Example

Be = 12.00

Le = -17.75

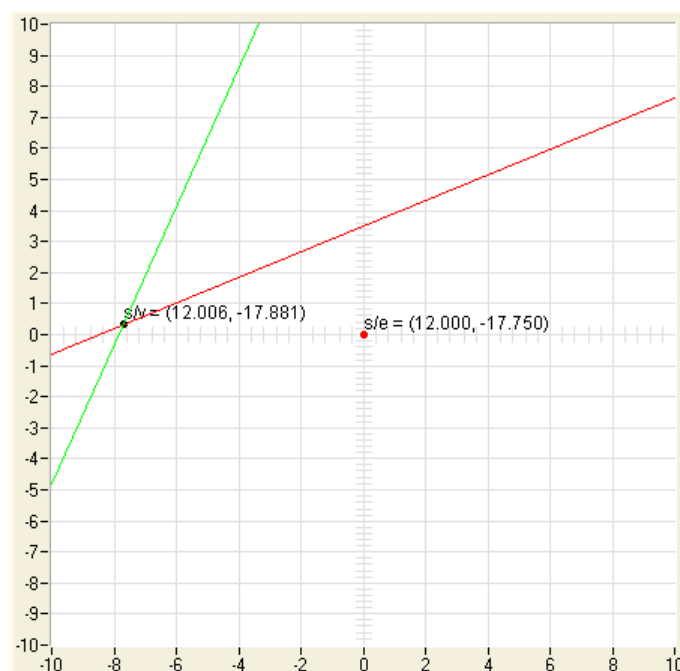
R = 343°

V = 0

Z (°)	p (mn)
337.5854	3.24
294.1474	7.152

B = 12.0056°

L = -17.8810°



## A3. Software

**Almanaque Nautico**

Efemerides El Sol La Luna Navegacion Astronomica Rumbos Ayuda

Fecha: 05/02/2009 UT1 = 9:28:14 Ahora

Cuerpo Celeste: ☒ Sol ☐ Planeta ☐ Luna ☐ Estrella ☐ Aries

**Posicion por dos Rectas de Altura**

Posicion Asumida en 1: Be = 43.31666° Le = -2°

Rectas Altura: 1 2

Z = 0 0° p = 0 0'

Derrota entre las dos RA: R = 0° d = 0 millas nauticas

Calcular Borrar

Calcula T [m] illum

Situacion: s/e

Observacion: Hs = 0° Parametros

Reconocimiento: Cielo visible Hc Z

Navegacion: Recta Altura Insertar tmp Posicion

Available at the Navigational Algorithms web page.

## A4. Source code

---

```

/*
FILE: 2RA.c

Posición por 2 Rectas de Altura

This file contains proprietary information of Andrés Ruiz Gonzalez
Andrés Ruiz. San Sebastian - Donostia. Gipuzkoa
Copyright (c) 1998
*/

#include <math.h>
#include "../Nav/angulos.hpp"

void Posicion2LOPSimultaneas( double Lat, double Lon, double p1, double Z1, double p2, double Z2,
                             double *B, double *L )
{
    double x, y; // [nm]

    *B = *L = 0;

    double a = SIN(Z1);
    double b = COS(Z1);
    double c = SIN(Z2);
    double d = COS(Z2);

    x = (p1*d-p2*b)/(a*d-b*c);
    y = (p2*a-p1*c)/(a*d-b*c);

    *B = Lat + y/60.0;
    *L = Lon + x/60.0/COS(*B);

    if ( *L > +180.0 ) *L -= 360.0;
    if ( *L < -180.0 ) *L += 360.0;
}

void Posicion2LOPNoSimultaneas ( double Lat, double Lon, double p1, double Z1, double p2, double Z2,
                                  double R, double D,
                                  double *B, double *L )
{
    Posicion2LOP( Lat, Lon, p1+D*COS(R-Z1), Z1, p2, Z2, B, L );
}

```

## A5. References

---

- <http://es.wikipedia.org/wiki/Recta>